FULL WAVE CHARACTERIZATION OF MICROSTRIP OPEN END DISCONTINUITIES PATTERNED ON ANISOTROPIC SUBSTRATES USING POTENTIAL THEORY

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Abstract

A technique for the full wave characterization of microstrip open end discontinuities fabricated on uniaxial anisotropic substrates using potential theory is presented. The substrate to be analyzed is enclosed in a cut-off waveguide, with the anisotropic axis aligned perpendicular to the air-dielectric interface. A full description of the sources on the microstrip line is included with edge conditions built in. Extention to other discontinuities is discussed.

Introduction

While there is extensive data available on the microwave characterization of a variety of microstrip discontinuities using both quasi-static [1-3] and full wave techniques [4-6], the characterization has been restricted to isotropic substrates. To date, there is no published data regarding microstrip discontinuities patterned on anisotropic substrates. Some very useful microwave substrates however, like sapphire, are anisotropic and so any discontinuity structures fabricated on them can not be properly characterized by the techniques that have been developed for isotropic dielectric substrates.

A technique for the full wave characterization of microstrip open ends fabricated on lossless uniaxial anisotropic substrates has been developed and is presented here. It is based on a dynamic source reversal technique that uses potential theory [7], which is a generalization of the charge resversal technique introduced several years ago [1]. The discontinuity is enclosed in a waveguide of infinite extent whose dimensions are such that the guide is cut-off for the propagating frequency on the microstrip. All sources on the microstrip are represented, and the technique does not require a model for the source exitation.

Dynamic Source Reversal Technique

The anisotropic axis of the substrate is aligned perpendicular to the air-dielectric interface as shown in Fig. 1. The anisotropic dielectric may be represented as a tensor quantity given by

$$\boldsymbol{\kappa}(y) = \kappa(y)\mathbf{I} + [\kappa_y(y) - \kappa(y)]\mathbf{a}_y\mathbf{a}_y \qquad (1)$$

where $\mathbf{I}$ is the unit dyad and $\kappa(y) = \kappa_y(y) = 1$ for $y > h$. The microstrip line is assumed to be infinitely thin and located at a height $y = h^+$. In terms of the sources on the microstrip line, the scalar and vector potentials, Φ and $\mathbf{A}$ respectively, for the dielectric loaded waveguide may be determined from

$$(\nabla^2 + \kappa k_0{}^2)A_x = -\mu_0 J_x \qquad (2)$$

$$(\nabla^2 + \kappa k_0{}^2)A_z = -\mu_0 J_z \qquad (3)$$

$$(\nabla^2 + \kappa_y k_0{}^2)A_y = j\omega\mu_0\epsilon_0(\kappa - 1)\Phi(h)\delta(y-h) + j\omega\mu_0\epsilon_0(\kappa_y - \kappa)\partial\Phi/\partial y \qquad (4)$$

$$[\kappa(\partial^2/\partial x^2 + \partial^2/\partial z^2) + \partial(\kappa(y)\partial/\partial y)/\partial y + \kappa^2(y)k_0{}^2]\Phi = -\rho/\epsilon_0 + j\omega(\kappa_y - 1)A_y(h)\delta(y-h) - j\omega(\kappa_y - \kappa)\partial A_y/\partial y \qquad (5)$$

The potentials appearing in Eqs. (2) to (5) are obtained from the appropriate Green's functions and the corresponding sources using

$$A_{x,z}(x,h,z) = \mu_0 \int_{x'}\int_{z'} G_{x,z}(x,h,z;x',h,z')J_{x,z}(x',h,z')dx'dz' \qquad (6)$$

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$$\epsilon_0\Phi(x,h,z) = \int_{x'}\int_{z'} G_{\Phi}(x,h,z;x',h,z')\rho(x',h,z')dx'dz' \quad (7)$$

From these potentials the electric field components are found using

$$E_x = -j\omega A_x - \partial\Phi/\partial x$$

$$E_y = -j\omega A_y - \partial\Phi/\partial y$$

$$E_z = -j\omega A_z - \partial\Phi/\partial z \quad (8)$$

where it is required that for this particular geometry E_x and E_z vanish on the microstrip. The fields thus obtained are expressed in terms of LSE and LSM modes of the dielectric loaded waveguide. The anisotropic effect appears only in the LSM mode terms, which are present in A_y and Φ. The LSE modes are unchanged from those obtained for the isotropic case.

A complete set of dominant mode sources on the microstrip are represented; the longitudinal and transverse currents, as well as the charge on the microstrip line, with appropriate edge conditions built in. For a wide range of practical open end discontinuities, a valid approximation is that $J_x = 0$, so therefore $A_x = 0$. As a result, only the boundary condition $E_z = 0$ is required for this problem.

A line terminated at $z = 0$, thus forming an open end, would create reflected dominant mode sources on the line, along with perturbed sources localized near the discontinuity. The total source distribution on an open end may be written as

$$J_z(x',h,z') = J_{0\,z}(x')(e^{-j\beta z'} - Re^{+j\beta z'}) + J_{1\,z}(x',z') \quad (9)$$

$$\rho(x',h,z') = \rho_0(x')(e^{-j\beta z'} + Re^{+j\beta z'}) + \rho_1(x',z') \quad (10)$$

for $z' \leq 0$, where $J_{0\,z}$, ρ_0 and β are the yet to be determined amplitudes and propagation constant, respectively of the dominant microstrip mode, R is an unknown reflection coefficient, and $J_{1\,z}$ and ρ_1 represent the perturbed source amplitudes near the open end. Weighted Chebychev polynomials are used to represent the sources in x for both dominant and perturbed sources, while triangle and pulse functions are used to represent the perturbed sources in z.

Equations (9) and (10) may be written as

$$J_z(x',h,z') = [j(1 + R)J_{0\,z}(x')(B_{in}\cos(\beta z') - \sin(\beta z')) + jJ_{1\,z}(x',z')] \cdot [1 - U(z')] \quad (11)$$

$$\rho(x',h,z') = [(1 + R)\rho_0(x')(B_{in}\sin(\beta z') + \cos(\beta z')) + \rho_1(x',z')] \cdot [1 - U(z')] \quad (12)$$

where $U(z')$ is the Heavyside unit step function which is 0 for $z' < 0$ or 1 for $z' > 0$, and $jB_{in} = (1 - R)/(1 + R)$ is the normalized input susceptance for the open end. In Eqs. (11) and (12) the dominant mode sources are assumed to exist for $-\infty \leq z' \leq \infty$. Then dominant mode sources for $z' > 0$ are subtracted away to create the terms that multiply the $(1 + R)$ coefficient in (11) and (12). Since the amplitudes of $J_{1\,z}$ and ρ_1 are arbitrary at this point, they may be defined so as to include the $(1 + R)$ term. Now the $(1 + R)$ term is common to all of the source terms, so it may be normalized to 1.0. Using Eqs. (11) and (12) in Eqs. (6) and (7) and substituting into Eq. (8) gives the electric field in terms of the sources on the open end microstrip line. When the requirement that $E_z = 0$ on the microstrip is enforced, the terms corresponding to the dominant mode on the infinite line already satisfy the boundary condition on the strip, so they drop out. The sources existing for $z' > 0$ may be considered "source reversed" terms which produce an impressed field in the reigon $z \leq 0$ but localized near the discontinuity. The factor $(1 + R)$ can be included into the arbitrary amplitude of the dominant mode sources. Thus, apart from the unknown parameter B_{in}, the dominant mode sources in $z' > 0$ produce a known forcing function in the strip for $z' < 0$. The electric field produced by the perturbed sources $J_{1\,z}$ and ρ_1, must cancel the tangential component of the applied field for $z \leq 0$. A modified perturbation technique [8] is used to determine the unknown dominant mode amplitudes and propagation constant for an infinite line. The method of moments is then used to reduce the resulting integral equation to a matrix equation which can be solved for the unknown input admittance B_{in}, as well as for $J_{1\,z}$ and ρ_1. Only one matrix inversion is required to find B_{in}.

Results

The pulse width, Δ, of the expansion functions was chosen to be 0.32 mm at f = 2.0 GHz (or $\Delta \simeq 0.0053\lambda_g$ for sapphire). This pulse width guaranteed a converged value for B_{in} [7] for all of the examples presented. To verify the accuracy of the theory as well as the resulting program, the program was checked for the isotropic case, and was able to duplicate data obtained in [1] and [7].

Table I shows the results obtained using this technique for several different anisotropic substrates as a function of microstrip line width. The open circuit capacitance, C_{oc} is found using $C_{oc} = B_{in}/\omega Z_0$, where the characteristic impedance Z_0 is obtained from a computer program developed on the basis of the theory presented in [8] for the characterization of infinite microstrip lines with sidewalls, but no top cover. To justify using values of Z_0 thus obtained, calculations performed for a microstrip line with an air dielectric showed less than 2% difference in Z_0 values obtained with and without a top cover. Table II shows the variation of C_{oc} as a function of line width for sapphire, and compares the results obtained for a substrate with an isotropic dielectric constant of 9.4, as well as for an isotropic dielectric constant of 11.6. Table III shows the effects of fixed waveguide dimensions on B_{in} and C_{oc} as a function of frequency of the propagating microstrip mode. For low frequencies, B_{in} varies linearly with frequency, starting to deviate as the frequency increases, this effect becomes more pronounced until the cut off frequency of the E_{11} waveguide mode is reached. This effect may be overcome by either frequency scaling the input parameters or by adjusting the waveguide dimensions accordingly.

The BASIC computer program developed to implement this technique can be executed on a personal computer with as little as 640K of RAM. Other discontinuity structures can be characterized in a similar manner. The technique is computationally efficient, there is no need to model the source excitation, and the admittance can be solved for directly in the case of a one port network. All integrals involving the expansion and testing functions are performed analytically so no numerical integrations are necessary, and the dominant portions of slowly converging series can be extracted and summed into closed form.

This technique can be extended to rapidly and accurately characterize a number of other commonly used discontinuity structures, especially "coaxial" two port structures such as asymmetrical gaps and steps in width. To characterize a two port structure in terms of an equivalent "Tee" or "Pi" network, the Tangent Plane method [9] can be used to extract parameter values.

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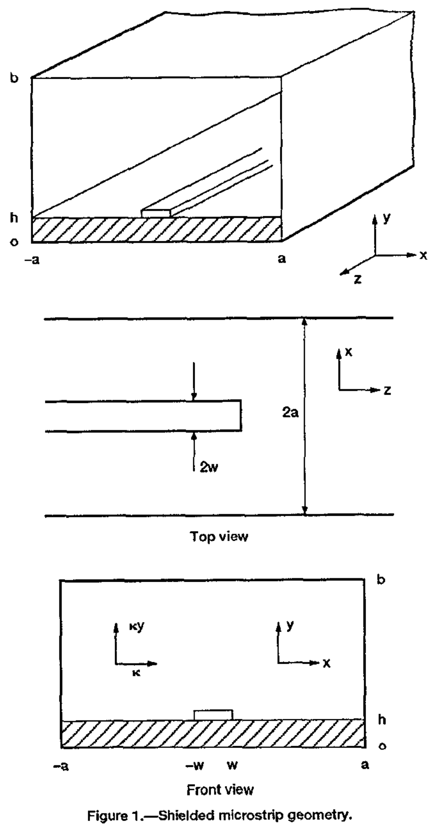

Figure 1.—Shielded microstrip geometry.

TABLE I.—OPEN CIRCUIT CAPACITANCE, C_{OC}, FOR SEVERAL ANISOTROPIC DIELECTRIC MATERIALS. FREQUENCY = 2.0 GHz, h = 1.0 mm, b = 11 mm, 2a = 20 mm FOR W/h < 4.0 ELSE 2a = 10(W/h). Z_0 VALUES OBTAINED FROM REF. 8. UNITS ARE pF/METER FOR C_{OC}.

		W/h					
		0.25	0.5	1.0	2.0	4.0	6.0
PTFE/Woven glass	κ_e	1.914	1.941	1.981	2.042	2.129	2.182
$\kappa = 2.84$	Z_0	150.5	119.9	90.02	62.46	39.82	29.50
$\kappa_y = 2.45$	C_{oc}/W	29.85	23.71	19.90	17.66	16.29	15.60
Boron nitride	κ_e	2.676	2.699	2.738	2.808	2.922	2.999
$\kappa = 5.12$	Z_0	127.4	101.7	76.68	53.38	34.09	25.25
$\kappa_y = 3.4$	C_{oc}/W	43.04	33.95	28.27	24.84	22.72	21.66
Sapphire	κ_e	6.724	7.012	7.647	8.145	9.007	9.514
$\kappa = 9.4$	Z_0	80.90	63.65	46.94	31.82	19.80	14.52
$\kappa_y = 11.6$	C_{oc}/W	80.36	65.56	56.38	50.95	47.43	45.35
Epsilam 10	κ_e	6.885	7.047	7.306	7.721	8.308	8.679
$\kappa = 13$	Z_0	79.90	63.44	47.39	32.61	20.55	15.14
$\kappa_y = 10.3$	C_{oc}/W	95.28	76.26	64.44	57.35	52.93	50.49

TABLE II.—VARIATION OF C_{OC} AS A FUNCTION OF LINE WIDTH FOR SAPPHIRE, k = 9.4, $k_y = 11.6$ COMPARED TO THAT OF AN ISOTROPIC DIELECTRIC. FREQUENCY = 2.0 GHz, h = 1.0 mm, b = 11 mm, 2a = 20 mm FOR W/h < 4.0, ELSE 2a = 10(W/h). UNITS ARE pF/METER FOR C_{OC}.

		W/h					
		0.25	0.5	1.0	2.0	4.0	6.0
Sapphire $\kappa = 9.4$ $\kappa_y = 11.6$	C_{oc}/W	80.36	65.56	56.38	50.95	47.43	45.35
$\kappa = 9.4$ $\kappa_y = 9.4$	C_{oc}/W	75.56	61.06	52.10	46.80	43.42	41.52
$\kappa = 11.6$ $\kappa_y = 11.6$	C_{oc}/W	90.18	73.27	62.58	56.21	52.13	49.77

TABLE III.—VARIATION OF THE NORMALIZED INPUT SUSCEPTANCE, B_{in}, AND OPEN CIRCUIT CAPACITANCE, C_{OC}, WITH FREQUENCY. WAVEGUIDE DIMENSIONS ARE 2a = 20 mm, b = 11 mm AND h = 1 mm.

Frequency, GHz	B_{in}	C_{oc}(pF/m)
0.5	0.008388	56.26
1.0	.016776	56.26
2.0	.033256	56.38
4.0	.068373	56.85
6.0	.167738	90.81